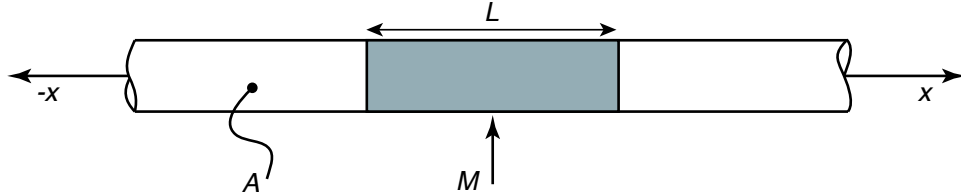


Exercise set 4

Problem 1



Consider the pipe section depicted in the figure above (similar to Problem 1 from the previous exercise set). A student injects 5 ml of 20% Rhodamine-WT solution (specific gravity $\frac{\rho_{\text{RhodamineWT}}}{\rho_{\text{H}_2\text{O}}} = 1.15$) instantaneously and uniformly over the pipe cross-section ($A = 0.8 \text{ cm}^2$) at the point $x = 0$ and the time $t = 0$. This time the injection process cannot be modeled by a point source, i.e. the volume of the injected solution must be taken into account. The pipe is filled with stagnant water. Assume the molecular diffusion coefficient is $D = 0.13 \cdot 10^{-4} \text{ cm}^2/\text{s}$.

- What is the concentration at $x = 0$ at the time $t = 0$?
- What is the standard deviation of the concentration distribution 1s after injection? Is the contribution of diffusion dominant? Discuss the validity of the point source approximation made in the Problem 1 of the previous exercise set.
- Consider an advection term with velocity v_{adv} in addition to the diffusion process (for example this could describe an intravenous injection). Compute the concentration distribution $\rho(x, t)$ in the presence of advection.
- Discuss the condition on v_{adv} for the advection to be negligible.

Hints:

- $\text{erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ (Exercise set 2)
- $\int_{-\infty}^{\infty} e^{-\alpha^2} d\alpha = \sqrt{\pi}$ (Exercise set 3)
- $\int_{-\infty}^{\infty} \alpha^2 e^{-\alpha^2} d\alpha = \frac{\sqrt{\pi}}{2}$ (Exercise set 3)